Chapter 2
Reasoning and Proof

Section 6
Proving Statements about Angles

GOAL 1: Congruence of Angles

In Lesson 2.5, you proved segment relationships. In this lesson you will prove statements about angles.

THEOREM

THEOREM 2.2 ***Properties of Angle Congruence***

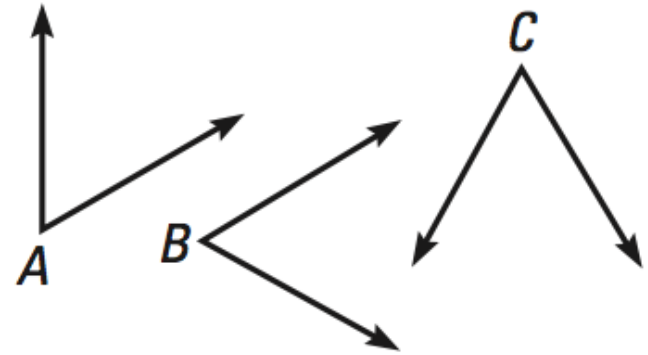
Angle congruence is reflexive, symmetric, and transitive.

Here are some examples.

REFLEXIVE	For any angle A, $\angle A \cong \angle A$.
SYMMETRIC	If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.
TRANSITIVE	If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

Example 1: Transitive Property of Angle Congruence

Prove the Transitive Property of Congruence for Angles.

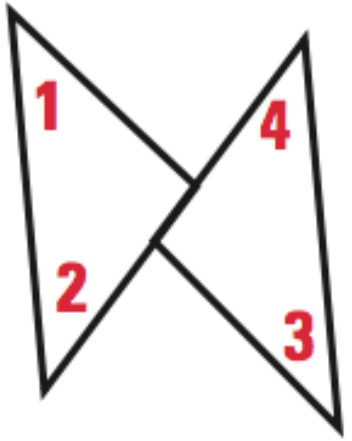


GIVEN ▶ $\angle A \cong \angle B$, $\angle B \cong \angle C$

PROVE ▶ $\angle A \cong \angle C$

Statements	Reasons
1. $\angle A \cong \angle B$, $\angle B \cong \angle C$	1. Given
2. $m\angle A = m\angle B$	2. Definition of congruent angles
3. $m\angle B = m\angle C$	3. Definition of congruent angles
4. $m\angle A = m\angle C$	4. Transitive property of equality
5. $\angle A \cong \angle C$	5. Definition of congruent angles

Example 2: Using the Transitive Property



This two-column proof uses the Transitive Property.

GIVEN ▶ $m\angle 3 = 40°, \angle 1 \cong \angle 2, \angle 2 \cong \angle 3$

PROVE ▶ $m\angle 1 = 40°$

Statements	Reasons
1. $m\angle 3 = 40°, \angle 1 \cong \angle 2, \angle 2 \cong \angle 3$	1. Given
2. $\angle 1 \cong \angle 3$	2. Transitive Property of Congruence
3. $m\angle 1 = m\angle 3$	3. Definition of congruent angles
4. $m\angle 1 = 40°$	4. Substitution property of equality

THEOREM

THEOREM 2.3 ***Right Angle Congruence Theorem***

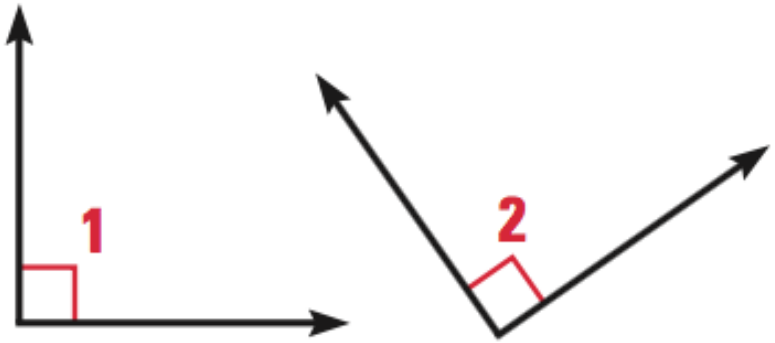
All right angles are congruent.

Example 3: Proving Theorem 2.3

You can prove Theorem 2.3 as shown.

GIVEN ▶ $\angle 1$ and $\angle 2$ are right angles

PROVE ▶ $\angle 1 \cong \angle 2$



Statements	Reasons
1. $\angle 1$ and $\angle 2$ are right angles	**1.** Given
2. $m\angle 1 = 90°$, $m\angle 2 = 90°$	**2.** Definition of right angle
3. $m\angle 1 = m\angle 2$	**3.** Transitive property of equality
4. $\angle 1 \cong \angle 2$	**4.** Definition of congruent angles

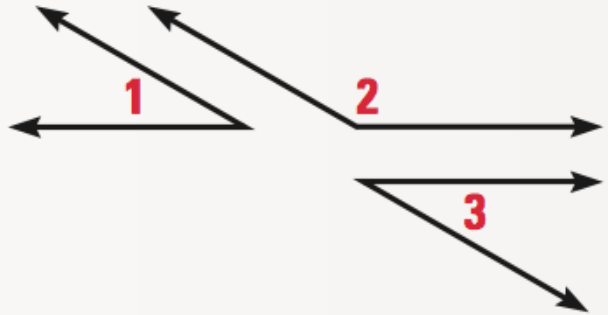
GOAL 2: Properties of Special Pairs of Angles

THEOREMS

THEOREM 2.4 *Congruent Supplements Theorem*

If two angles are supplementary to the same angle (or to congruent angles) then they are congruent.

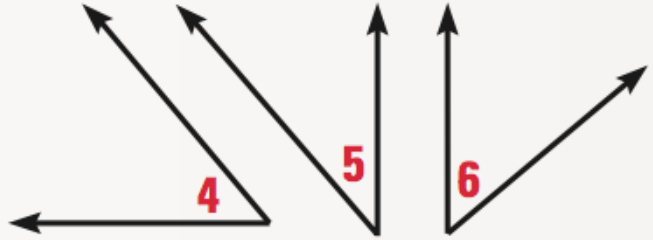
If $m\angle 1 + m\angle 2 = 180°$ and $m\angle 2 + m\angle 3 = 180°$, then $\angle 1 \cong \angle 3$.



THEOREM 2.5 *Congruent Complements Theorem*

If two angles are complementary to the same angle (or to congruent angles) then the two angles are congruent.

If $m\angle 4 + m\angle 5 = 90°$ and $m\angle 5 + m\angle 6 = 90°$, then $\angle 4 \cong \angle 6$.



Example 4: Proving Theorem 2.4

GIVEN ▶ $\angle 1$ and $\angle 2$ are supplements,
$\angle 3$ and $\angle 4$ are supplements,
$\angle 1 \cong \angle 4$

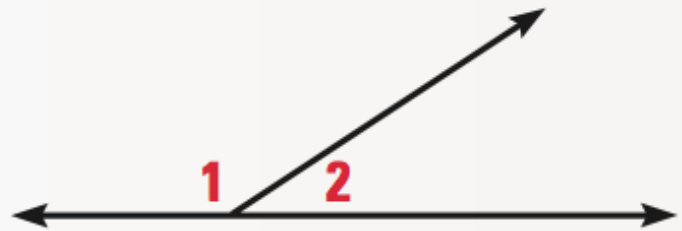
PROVE ▶ $\angle 2 \cong \angle 3$

Statements	Reasons
1. $\angle 1$ and $\angle 2$ are supplements, $\angle 3$ and $\angle 4$ are supplements, $\angle 1 \cong \angle 4$	**1.** Given
2. $m\angle 1 + m\angle 2 = 180°$ $m\angle 3 + m\angle 4 = 180°$	**2.** Definition of supplementary angles
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	**3.** Transitive property of equality
4. $m\angle 1 = m\angle 4$	**4.** Definition of congruent angles
5. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$	**5.** Substitution property of equality
6. $m\angle 2 = m\angle 3$	**6.** Subtraction property of equality
7. $\angle 2 \cong \angle 3$	**7.** Definition of congruent angles

POSTULATE

POSTULATE 12 ***Linear Pair Postulate***

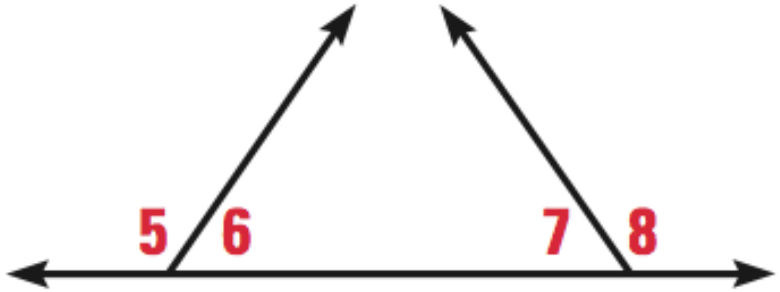
If two angles form a linear pair, then they are supplementary.



$m\angle 1 + m\angle 2 = 180°$

Example 5: Using Linear Pairs

In the diagram, $m\angle 8 = m\angle 5$ and $m\angle 5 = 125°$.
Explain how to show $m\angle 7 = 55°$.



m<8 = m<5 & m<5 = 125* → m<8 = 125*

<7 & <8 → linear pair → 180 – 125 = 55 → m<7 = 55*

THEOREM

THEOREM 2.6 ***Vertical Angles Theorem***

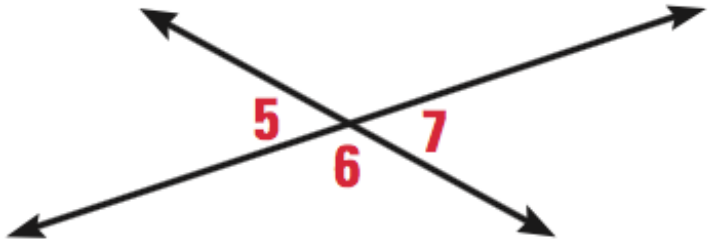
Vertical angles are congruent.

$\angle 1 \cong \angle 3, \angle 2 \cong \angle 4$

Example 6: Proving Theorem 2.6

GIVEN ▶ $\angle 5$ and $\angle 6$ are a linear pair,
$\angle 6$ and $\angle 7$ are a linear pair

PROVE ▶ $\angle 5 \cong \angle 7$



Statements	Reasons
1. $\angle 5$ and $\angle 6$ are a linear pair, $\angle 6$ and $\angle 7$ are a linear pair	**1.** Given
2. $\angle 5$ and $\angle 6$ are supplementary, $\angle 6$ and $\angle 7$ are supplementary	**2.** Linear Pair Postulate
3. $\angle 5 \cong \angle 7$	**3.** Congruent Supplements Theorem